XYZ to sRGB(D65)

Masayuki Tanaka

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1 Tristimulus XYZ

The XYZ is the quasi stimulus of the standard observer (human). These tristimulus are related to the long, medium, and short cones [1]. The tristimulus XYZ can be calculated with color matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ as [2]

$$X = \int I(\lambda)\bar{x}(\lambda)\, d\lambda\,, \quad (1)$$

$$Y = \int I(\lambda)\bar{y}(\lambda)\, d\lambda\,, \quad (2)$$

$$Z = \int I(\lambda)\bar{z}(\lambda)\, d\lambda\,. \quad (3)$$

where $I(\lambda)$ is the irradiance.

The normalized tristimulus xyz is defined by

$$x = \frac{X}{X+Y+Z}\,, \quad (4)$$

$$y = \frac{Y}{X+Y+Z}\,, \quad (5)$$

$$z = \frac{Z}{X+Y+Z} = 1 - x - y\,. \quad (6)$$

2 sRGB

The sRGB uses the primaries in the ITU-R BT.709. The chromaticities of the red, green, and blue primaries are defined as [3]

(R,G,B)	(x,y,z)
(1,0,0)	(0.64, 0.33, 0.03)
(0,1,0)	(0.30, 0.60, 0.10)
(0,0,1)	(0.15, 0.06, 0.79)

The light source of (R,G,B)=(1,0,0) is defined by the ligh source which gives the normalized tristrimulus (x,y,z)=(0.64, 0.33, 0.03).

The sRGB is usually applied gamma. Here, (R, G, B) and (R_s, G_s, B_s) refer the linear sRGB and the sRGB.

$$R_s = R^{1/2.4}\,, \quad (7)$$

$$G_s = G^{1/2.4}\,, \quad (8)$$

$$B_s = B^{1/2.4}\,. \quad (9)$$

$$(10)$$

$$R = R_s^{2.4}\,, \quad (11)$$

$$
\begin{aligned}
G &= G_s^{2.4}\,, && (12)\\
B &= B_s^{2.4}\,. && (13)\\
& && (14)
\end{aligned}
$$

3 White point

$$
\begin{aligned}
X &= \int r(\lambda)L(\lambda)\bar{x}(\lambda)\,d\lambda\,, && (15)\\
Y &= \int r(\lambda)L(\lambda)\bar{y}(\lambda)\,d\lambda\,, && (16)\\
Z &= \int r(\lambda)L(\lambda)\bar{z}(\lambda)\,d\lambda\,. && (17)
\end{aligned}
$$

where $r(\lambda)$ is the referectance and the $L(\lambda)$ is the lighting.

The white point is here defined for each lighting by the tristrimulus for the perfect white object $r(\lambda) = 1$, namly

$$
\begin{aligned}
X'_w &= \int L(\lambda)\bar{x}(\lambda)\,d\lambda\,, && (18)\\
Y'_w &= \int L(\lambda)\bar{y}(\lambda)\,d\lambda\,, && (19)\\
Z'_w &= \int L(\lambda)\bar{z}(\lambda)\,d\lambda\,. && (20)
\end{aligned}
$$

This white point has an scale ambiguity. To avoid this scale ambiguity, the white point is usually normalized by the Y as

$$
\begin{aligned}
X_w &= X'_w/Y'_w\,, && (21)\\
Y_w &= Y'_w/Y'_w = 1\,, && (22)\\
Z_w &= Z'_w/Y'_w\,, && (23)\\
& && (24)
\end{aligned}
$$

For example, the white point of the D65 light is

$$
(X_w, Y_w, Z_w) = (0.950, 1.000, 1.089) \tag{25}
$$

4 XYZ to sRGB (D65)

The linear sRGB is mapped to the normalized tristimulus as

$$
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.64 & 0.30 & 0.15 \\ 0.33 & 0.60 & 0.10 \\ 0.03 & 0.10 & 0.79 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} = \boldsymbol{m} \begin{pmatrix} R \\ G \\ B \end{pmatrix}\,. \tag{26}
$$

The red light source of (R,G,B) = (1,0,0) gives the following tristimulus.

$$
\begin{pmatrix} 0.64 \\ 0.33 \\ 0.03 \end{pmatrix} R = \begin{pmatrix} x_R \\ y_R \\ z_R \end{pmatrix} = \begin{pmatrix} X_R/(X_R+Y_R+Z_R) \\ Y_R/(X_R+Y_R+Z_R) \\ Z_R/(X_R+Y_R+Z_R) \end{pmatrix}\,. \tag{27}
$$

This equation can be written as

$$
\begin{pmatrix} 0.64 \\ 0.33 \\ 0.03 \end{pmatrix} T_R R = \begin{pmatrix} X_R \\ Y_R \\ Z_r \end{pmatrix}\,, \tag{28}
$$

where $T_R = X_R + Y_R + Z_R$. The parameter T_R corresponds the gain of the display device. For the G and B light sources, we have same relation.

Then, the tristimulus XYZ for the given sRGB is expressed as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \boldsymbol{m} \begin{pmatrix} T_R R \\ T_G G \\ T_B B \end{pmatrix} = \boldsymbol{m}\mathrm{diag}(T_R, T_G, T_B) \begin{pmatrix} R \\ G \\ B \end{pmatrix}, \tag{29}$$

where T_G and T_B are the scale parameter for the G and B light sources.

The matrix $\boldsymbol{m}$ is known by the definition. The scale parameters T_R, T_G, T_B depend on the display device. To avoid this display device dependency, the scale parameters are determined assuming the lighting condition or the white point. The scale factors T_R, T_G, T_B are set, so that the white point tristimulus are received when $(R, G, B) = (1, 1, 1)$. The white point depends on the lighting. Therefore, the matrix which transform from XYZ to sRGB depend on the lighting.

$$\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = \boldsymbol{m} \begin{pmatrix} T_R \\ T_G \\ T_B \end{pmatrix} \tag{30}$$

$$\begin{pmatrix} T_R \\ T_G \\ T_B \end{pmatrix} = \boldsymbol{m}^{-1} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} = \begin{pmatrix} 0.64 & 0.30 & 0.15 \\ 0.33 & 0.60 & 0.06 \\ 0.03 & 0.10 & 0.79 \end{pmatrix}^{-1} \begin{pmatrix} 0.950 \\ 1.000 \\ 1.089 \end{pmatrix} = \begin{pmatrix} 0.6444 \\ 1.1919 \\ 1.2032 \end{pmatrix} \tag{31}$$

$$\boldsymbol{m}\,\mathrm{diag}(T_R, T_G, T_B) = \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix} \tag{32}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \boldsymbol{m}\,\mathrm{diag}(T_R, T_G, T_B) \begin{pmatrix} R \\ G \\ B \end{pmatrix} == \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \{\boldsymbol{m}\,\mathrm{diag}(T_R, T_G, T_B)\}^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{34}$$

References

[1] Henry R. Kang, Computaonal Color Technology, SPIE press, 2006.

[2] http://en.wikipedia.org/wiki/LMS_color_space

[3] http://en.wikipedia.org/wiki/CIE_1931_color_space

[4] https://en.wikipedia.org/wiki/SRGB